Matematika: od zapletene simbolnosti do preproste vsakdanjosti

Mathematics: from symbolic complexity to everyday simplicity

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Doing the Math to Find the Good Jobs

Mathematicians Land Top Spot in New Ranking of Best and Worst Occupations in the U.S.

http://online.wsj.com/article/SB123119236117055127.html

The Best

1. Mathematician
2. Actuary
3. Statistician
4. Biologist
5. Software Engineer
6. Computer Systems Analyst

The Worst

1. Lumberjack
2. Dairy Farmer
3. Taxi Driver
4. Seaman
5. EMT
6. Roofer

Scott Brundage

Matematično ogrevanje
Mathematics Warm Ups

1960s :

A peasant sells a bag of potatoes for 10 dollars. His costs amount to 4/5 of his selling price. What is his profit?

1970s :

A farmer sells a bag of potatoes for 10 dollars. His costs amount to 4/5 of his selling price, that is, 8. What is his profit?

1970s (new math):

A farmer exchanges a set P of potatoes with set M of money. The cardinality of the set M is equal to 10, and each element of M is worth 1 dollar. Draw ten big dots representing the elements of M. The set C of production costs is composed of two big dots less than the set M. Represent C as a subset of M and give the answer to the question: What is the cardinality of the set of profits?

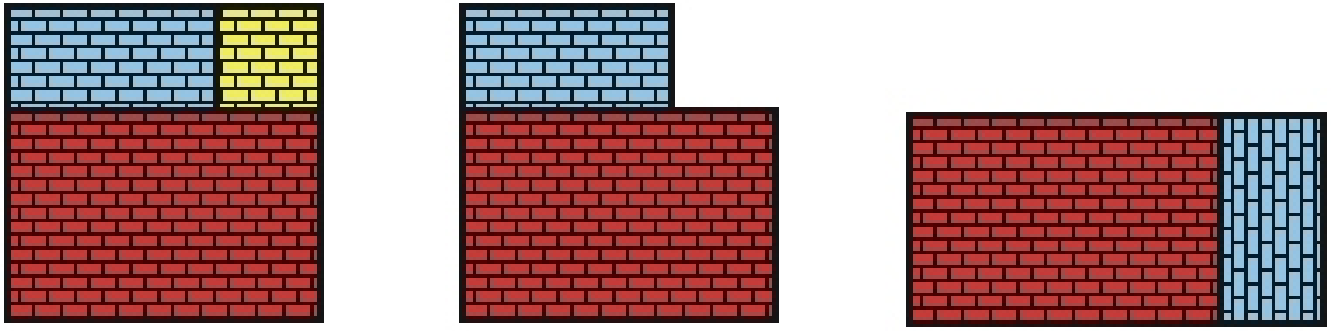
1980s :

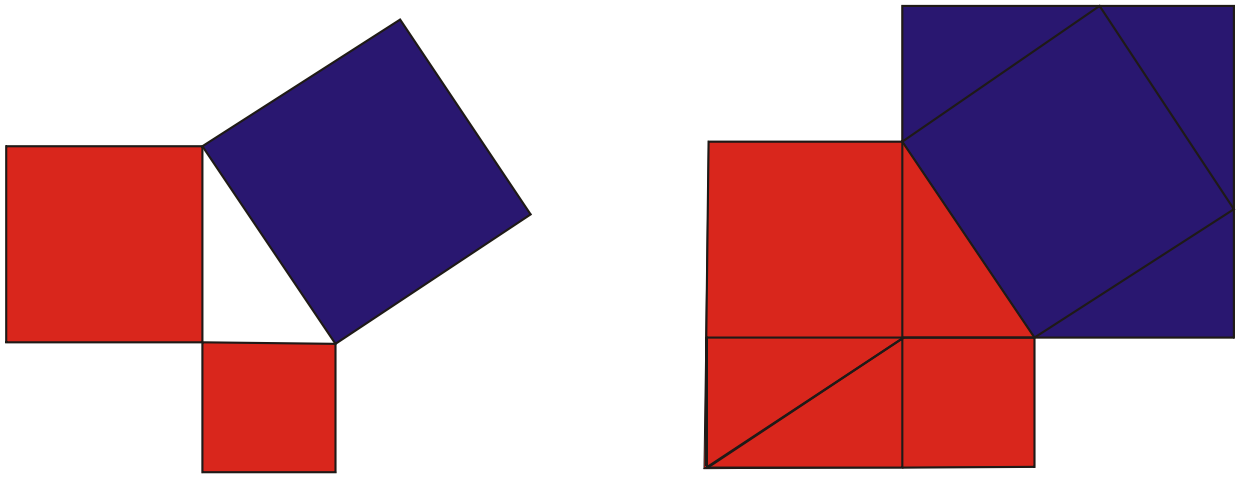
A farmer sells a bag of potatoes for 10 dollars. His production costs are 8 dollars, and his profit is 2 dollars. Underline the word "potatoes" and discuss with your classmates.

1990s through the new millennium:

A farmer sells a bag of potatoes for 10 dollars. His or her production costs are 0.80 of his or her revenue. On your calculator, graph revenue vs. costs. Run the POTATO program to determine the profit. Discuss the result with students in your group. Write a brief essay that analyzes this example in the world of global economics.

$x^2 - y^2 = (x + y)(x - y)$





Množenje s številom 11

- Preprosto!

  $25x11 = 275$
  $31x11 = 341$
  $57x11 = 627 \rightarrow 1$ se prenese!

- Za množenje dvomestnega števila z 11 seštej njegovi števki in njuno vsoto vstavi med njiju.

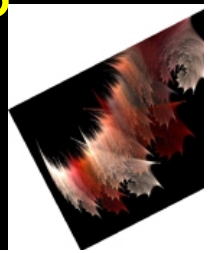
- Kaj pa 3-mestno število?

  $117x11 = 1287$
  $532x11 = 5852$
  $267x11 = 2937$

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$
$$12345 \times 8 + 5 = 987\ 65$$
$$123456 \times 8 + 6 = 987654$$
$$1234567 \times 8 + 7 = 9876543$$
$$12345678 \times 8 + 8 = 98765432$$
$$123456789 \times 8 + 9 = 987654321$$

$$1 \times 9 + 2 = 11$$
$$12 \times 9 + 3 = 111$$
$$123 \times 9 + 4 = 1111$$
$$1234 \times 9 + 5 = 11111$$
$$12345 \times 9 + 6 = 111111$$
$$123456 \times 9 + 7 = 1111111$$
$$1234567 \times 9 + 8 = 11111111$$
$$12345678 \times 9 + 9 = 111111111$$
$$123456789 \times 9 + 10 = 1111111111$$

$$9 \times 9 + 7 = 88$$
$$98 \times 9 + 6 = 888$$
$$987 \times 9 + 5 = 8888$$
$$9876 \times 9 + 4 = 88888$$
$$98765 \times 9 + 3 = 888888$$
$$987654 \times 9 + 2 = 8888888$$
$$9876543 \times 9 + 1 = 88888888$$
$$98765432 \times 9 + 0 = 888888888$$



1 x 1 = 1

11 x 11 = 121

111 x 111 = 12321

1111 x 1111 = 1234321

11111 x 11111 = 123454321

111111 x 111111 = 12345654321

1111111 x 1111111 = 1234567654321

11111111 x 11111111 =

123456787654321

111111111 x 111111111 =

12345678987654321

Črtne kode ali zakaj včasih blagajniški čitalec zapiska

Barcodes

Vsaka števka je predstavljena z zaporedjem belih in črnih črtic, ki odslikujejo binarni zapis števila (0 - bela črtica, 1 - črna črtica).

Kontrolna številka

Država Proizvajalec Artikel Kontrolna številka

Kontrolna številka

Zakaj?

Odpravljanje napak. Do napak prihaja pri odčitavanju črtne kode, s pomočjo kontrolne številke pa lahko zaznamo večino napak in tako ponovimo odčitavanje.

Tip napake	Število	Napaka	Frekvenca
Napaka ene števke	7501031311309	7501031**6**11309	79,1%
Zamenjava sosedov	7501031311309	7501031**13**1309	10,2%
Drugo	7501031311309	*	10,7%

7501031311309

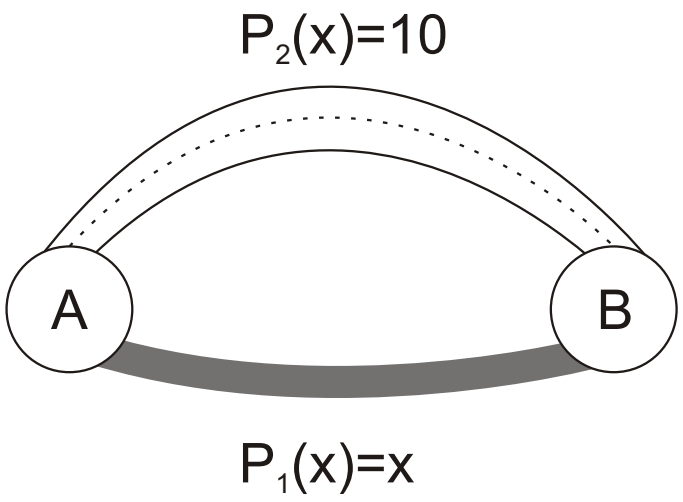
Kako?

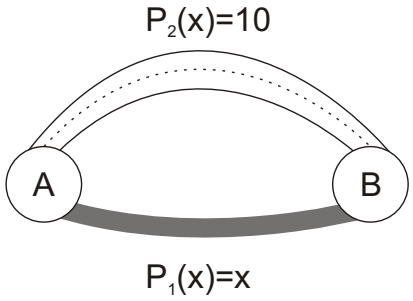
- Prvemu število določimo utež 1, drugemu 3, tretjemu spet 1, itd.
- Števila in uteži zmnožimo, zmnožke pa potem seštejemo.
- Dobljena vsota mora biti število deljivo z 10.

$$7 + 5 \cdot 3 + 0 + 1 \cdot 3 + 0 + 3 \cdot 3 + 1 + 3 \cdot 3 + 1 + 1 \cdot 3 + 3 + 0 \cdot 3 + 9 = 60$$

Pretočnost omrežij ali zakaj več ni vedno boljše kot manj

Network Flow

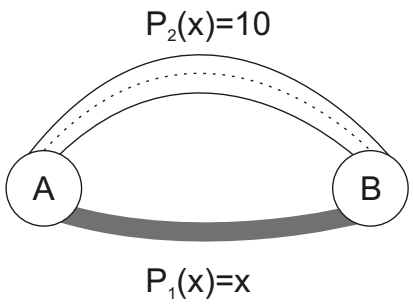

P2(x)=10
A
B
P1(x)=x



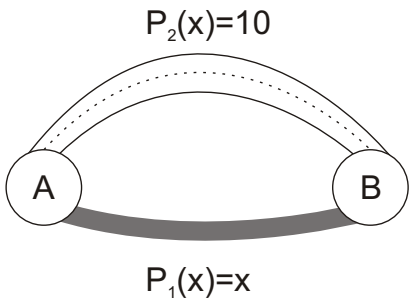
10 avtomobilov vozi iz A v B.

Po avtocesti (zgoraj) je čas, neodvisen od števila avtomobilov, vedno enak 10 minut.

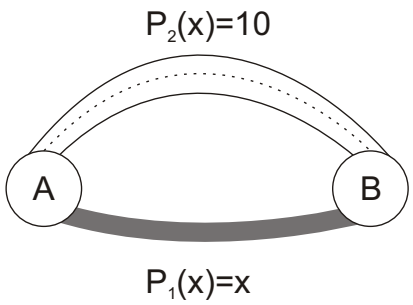
Po ulici (spodaj) je čas, odvisen od števila avtomobilov x, enak x minut.



Nash-evo ravnovesje je stanje, v katerem noben posameznik ne more izboljšati svoje situacije z drugačno izbiro (v tem primeru je to stanje, v katerem vsi izberejo ulico in je čas vožnje za vse sodelujoče enak 10 minut).

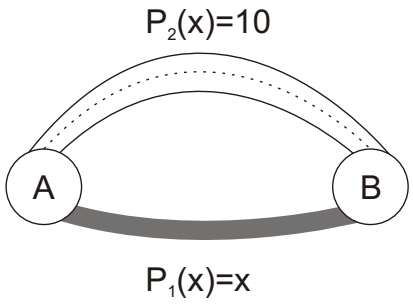


Nash-evo ravnovesje pa ni enako globalno optimalni rešitvi, to je, rešitvi, v kateri je skupno porabljeni čas najmanjši.



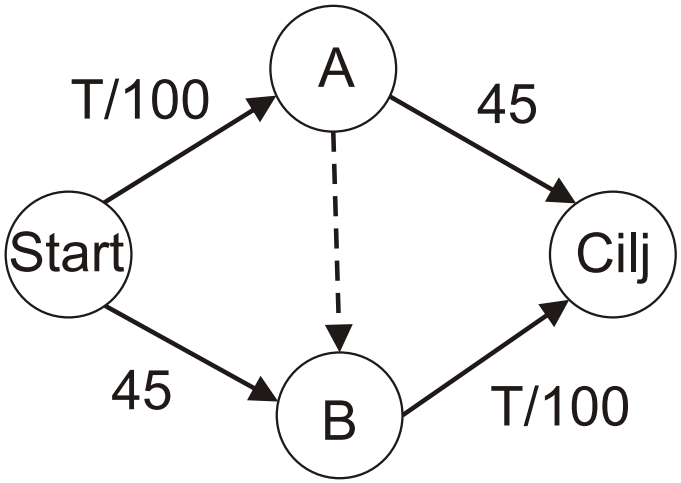
Povprečni čas vožnje:

$$
\begin{aligned}
P(x) &= \frac{xP_1(x) + (10-x)P_2(10-x)}{10} \\
&= \frac{x^2 + 10(10-x)}{10} \\
&= \frac{x^2}{10} - x + 10
\end{aligned}
$$

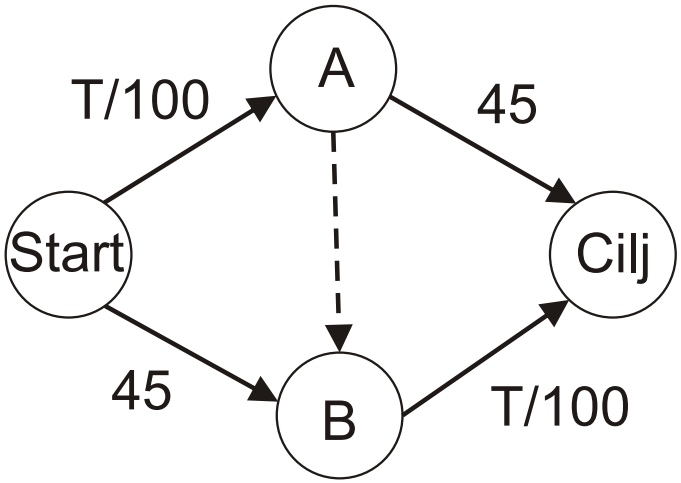


$$P(x) = \frac{x^2}{10} - x + 10$$

x	1	2	3	4	5	6	7	8	9	10
$P(x)$	9,1	8,4	7,9	7,6	7,5	7,6	7,9	8,4	9,1	10



- 4000 avtomobilov vozi iz starta na cilj.
- Če črtkana cesta ne obstaja, je čas potovanja po poti $Start - A - Cilj$ enak $A/100 + 45$ in čas potovanja po poti $Start - B - Cilj$ enak $B/100 + 45$, kjer je A število avtomobilov, ki vozijo po poti $Start - A - Cilj$ in B število avtomobilov, ki vozijo po poti $Start - B - Cilj$. V tem primeru nastopi Nash-evo ravnovesje takrat, ko polovica avtomobilov uporabi pot $Start - A - Cilj$ in polovico pot $Start - B - Cilj$. Čas vožnje je tedaj za vse enak $\frac{2000}{100} + 45 = 65$ minut.

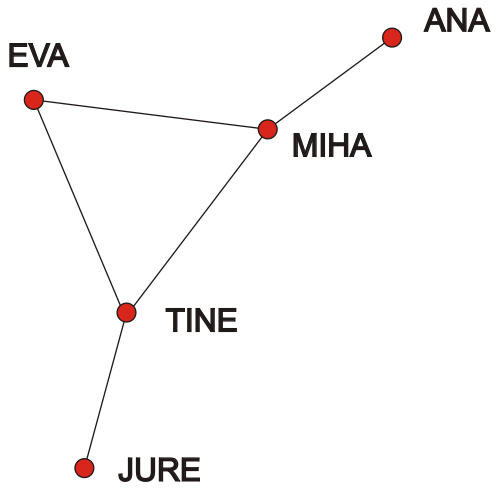


- Če dodamo črtkano cesto, po kateri vožnja traja 1 minuto, se bo čas vožnje povečal na 81 minut, saj je za vsakega posameznega voznika najoptimalneje uporabiti pot $Start - A - B - Cilj$.

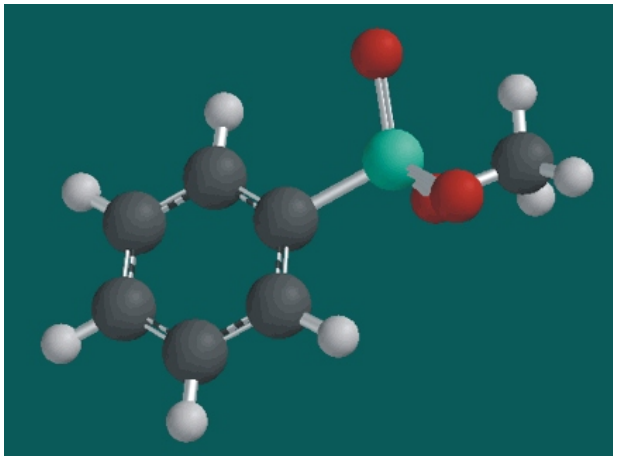
Grafi, simetrija, potovanja

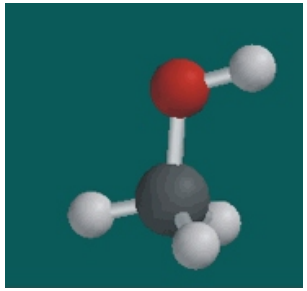
Graphs, Symmetry, Traversability

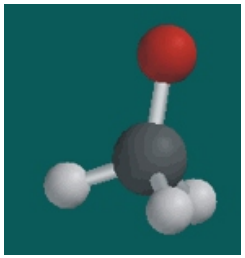
Graf lahko predstavimo z diagramom, ki ga sestavljajo točke in povezave.



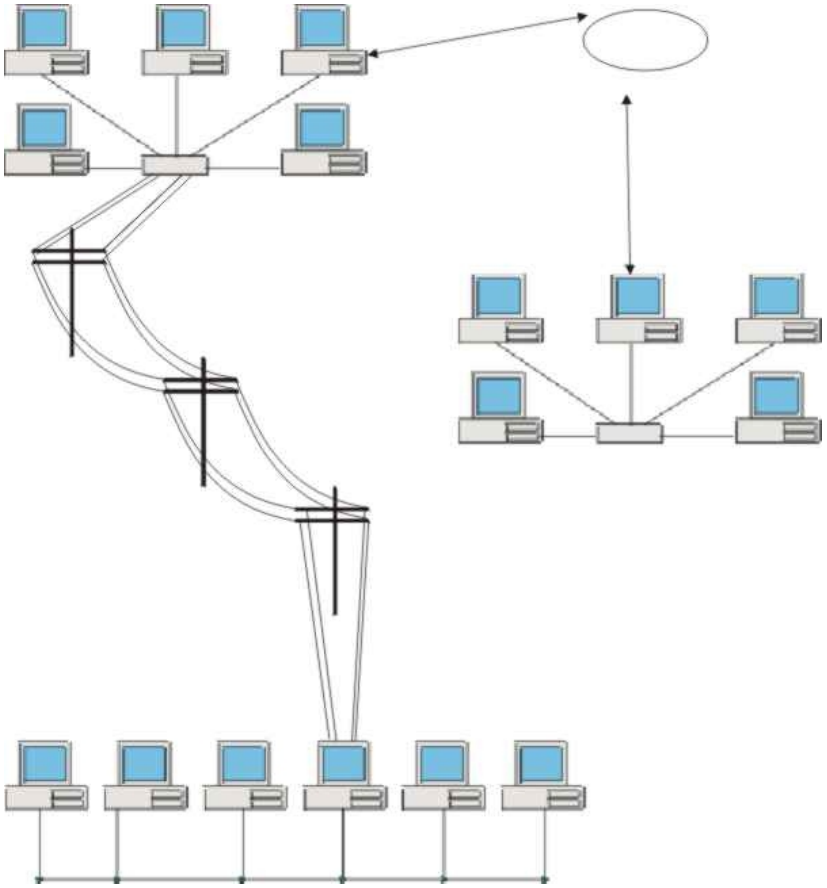
Molekularni grafi



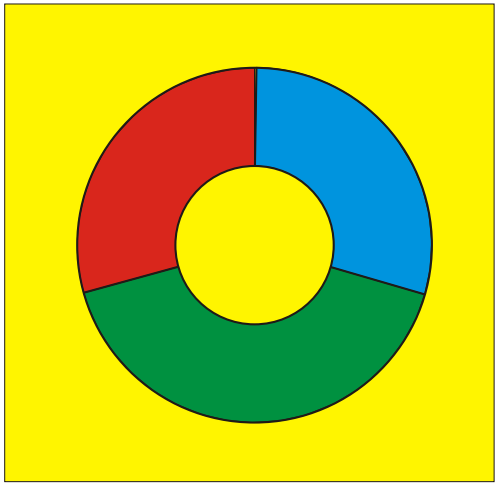




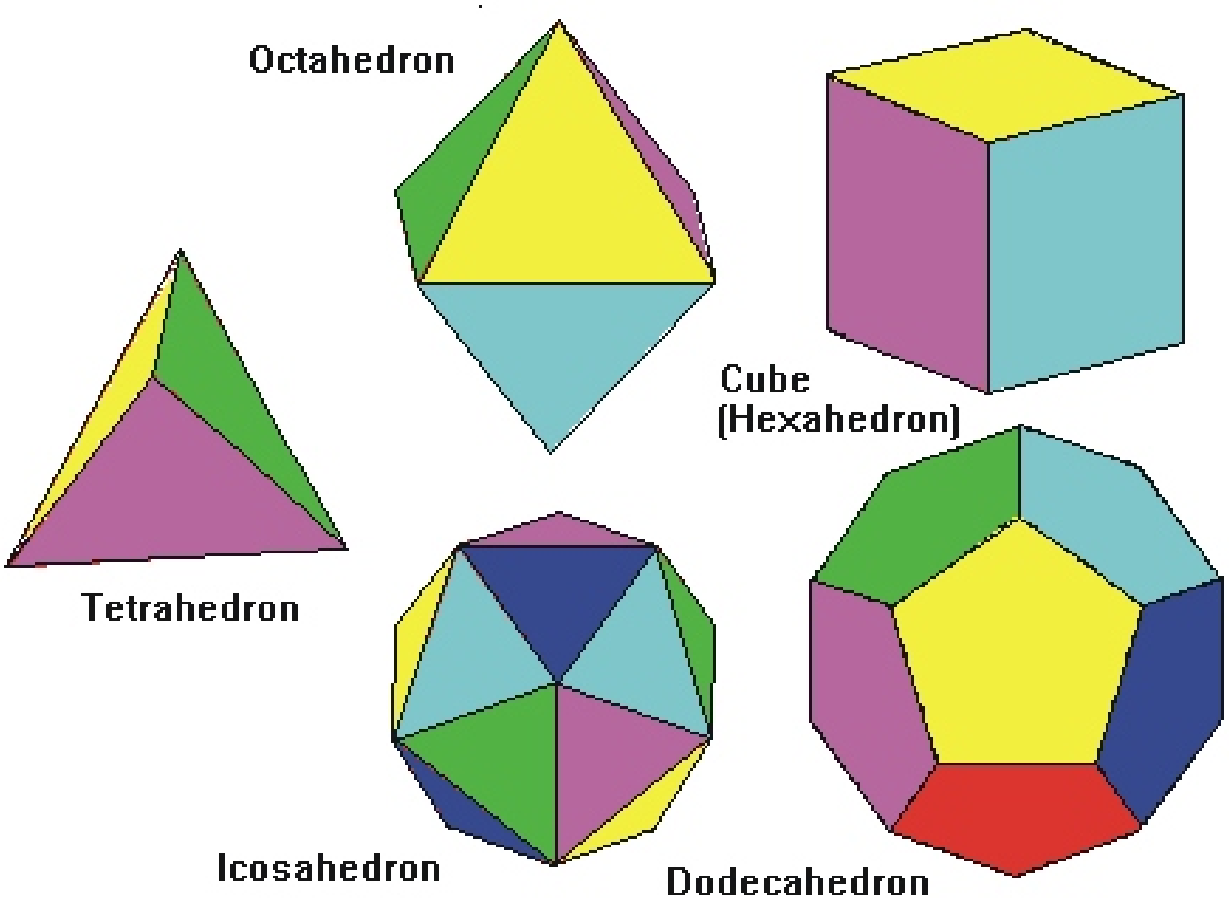
Računalniško omrežje



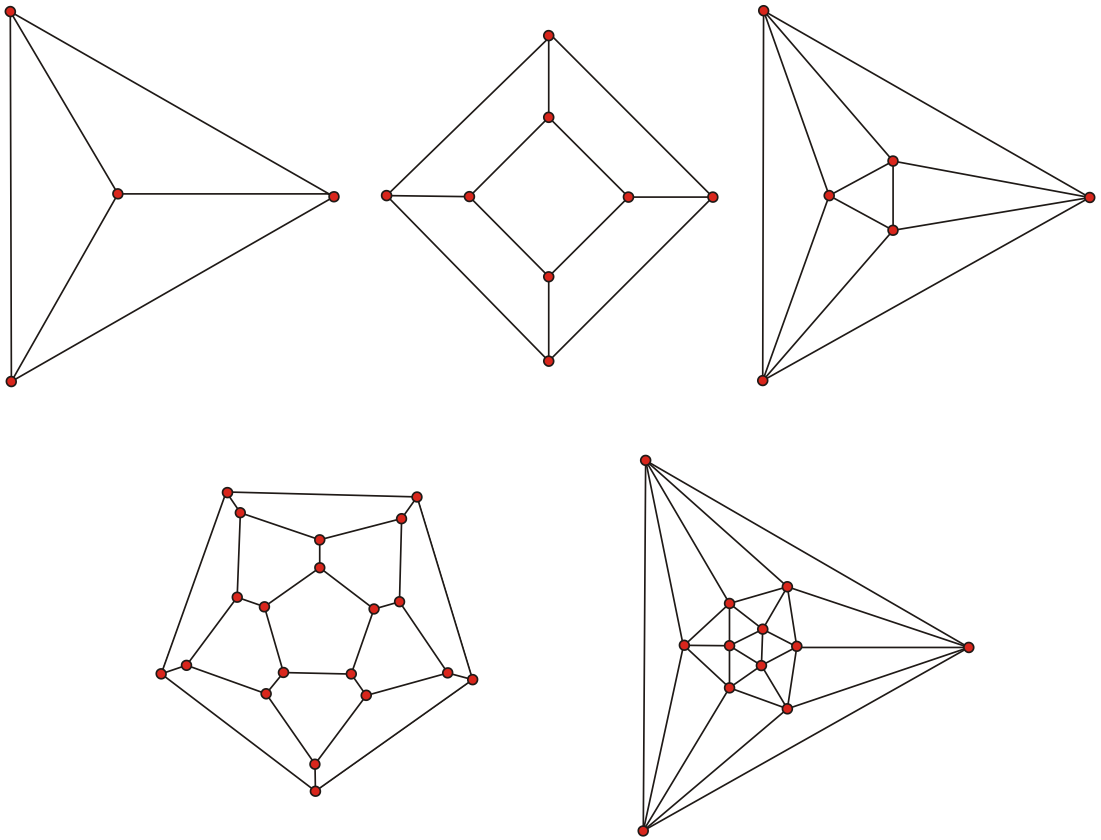
Problem štirih barv



Platonska telesa



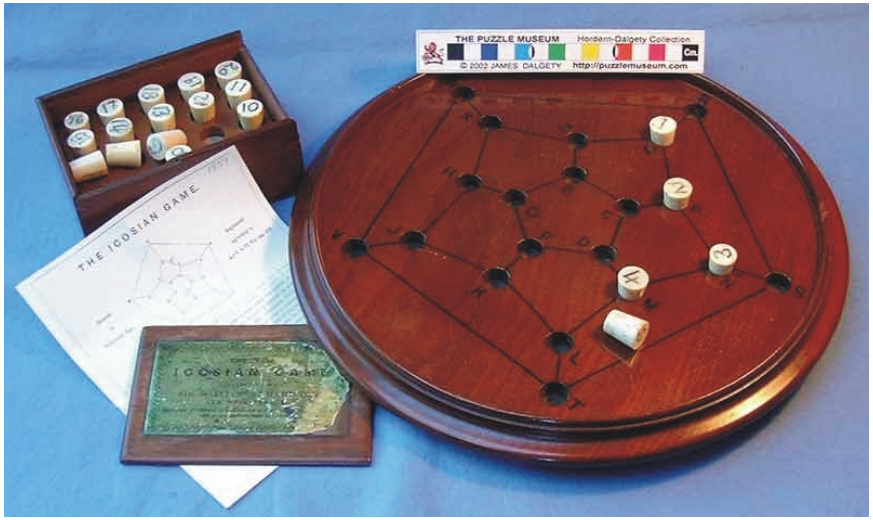
Platonska telesa



Problem trgovskega potnika

Hamiltonova Ikozaedrska igra (1857)

Ali obstaja cikel, ki prepotuje vsako točko natanko enkrat?

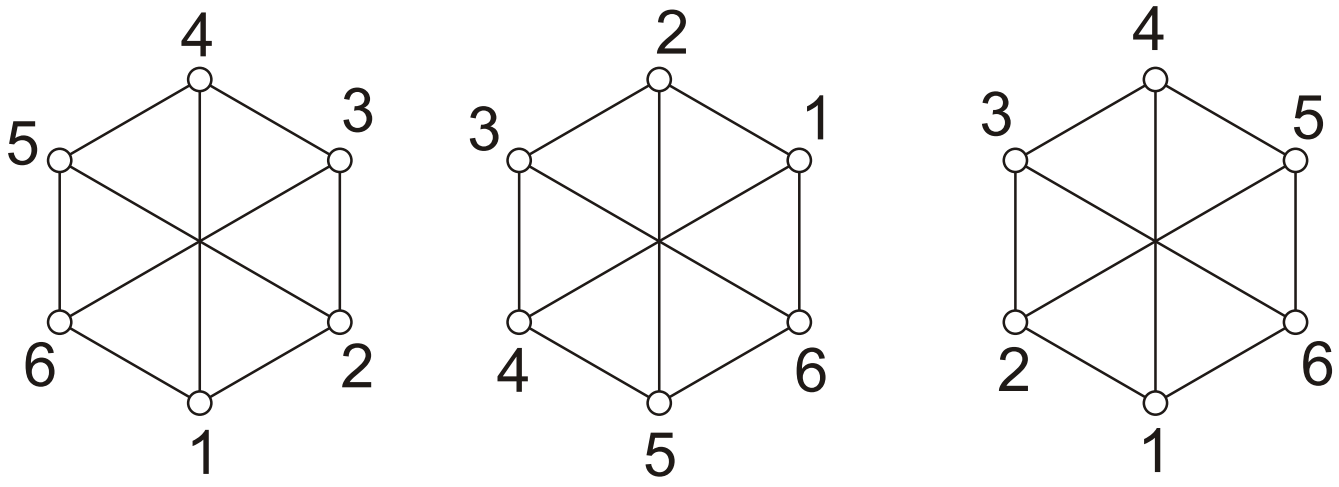


W. R. Hamilton in T. P. Kirkman

Hamiltonska pot (oziroma cikel)

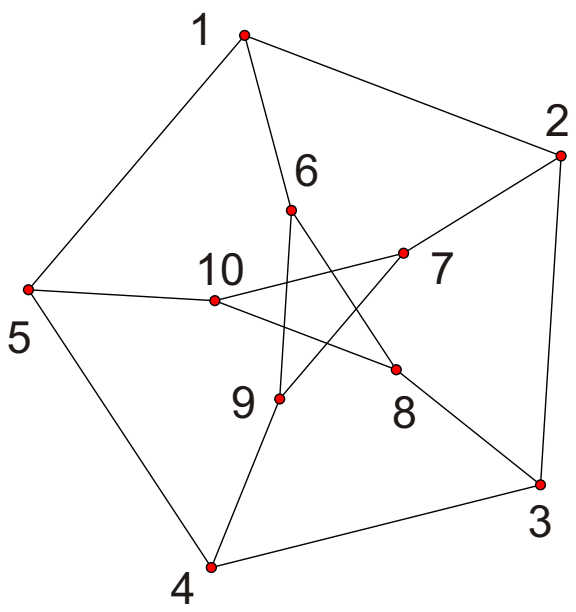
Pot (oziroma cikel) je hamiltonska, če gre skozi vsako točko natanko enkrat (če torej napenja cel graf).

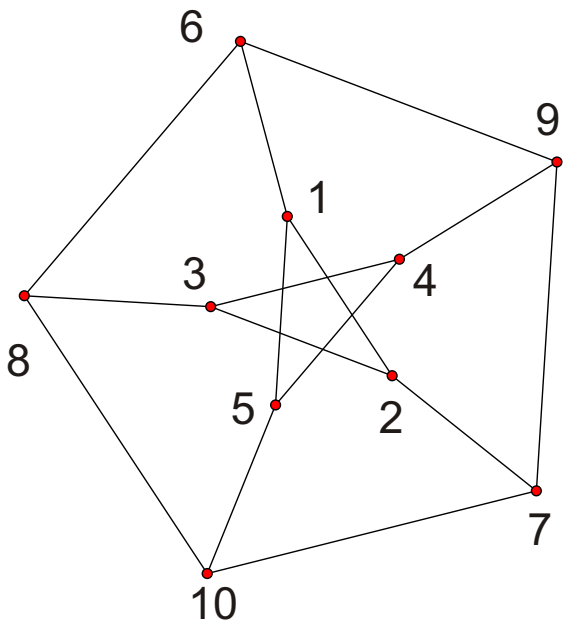
Simetrija



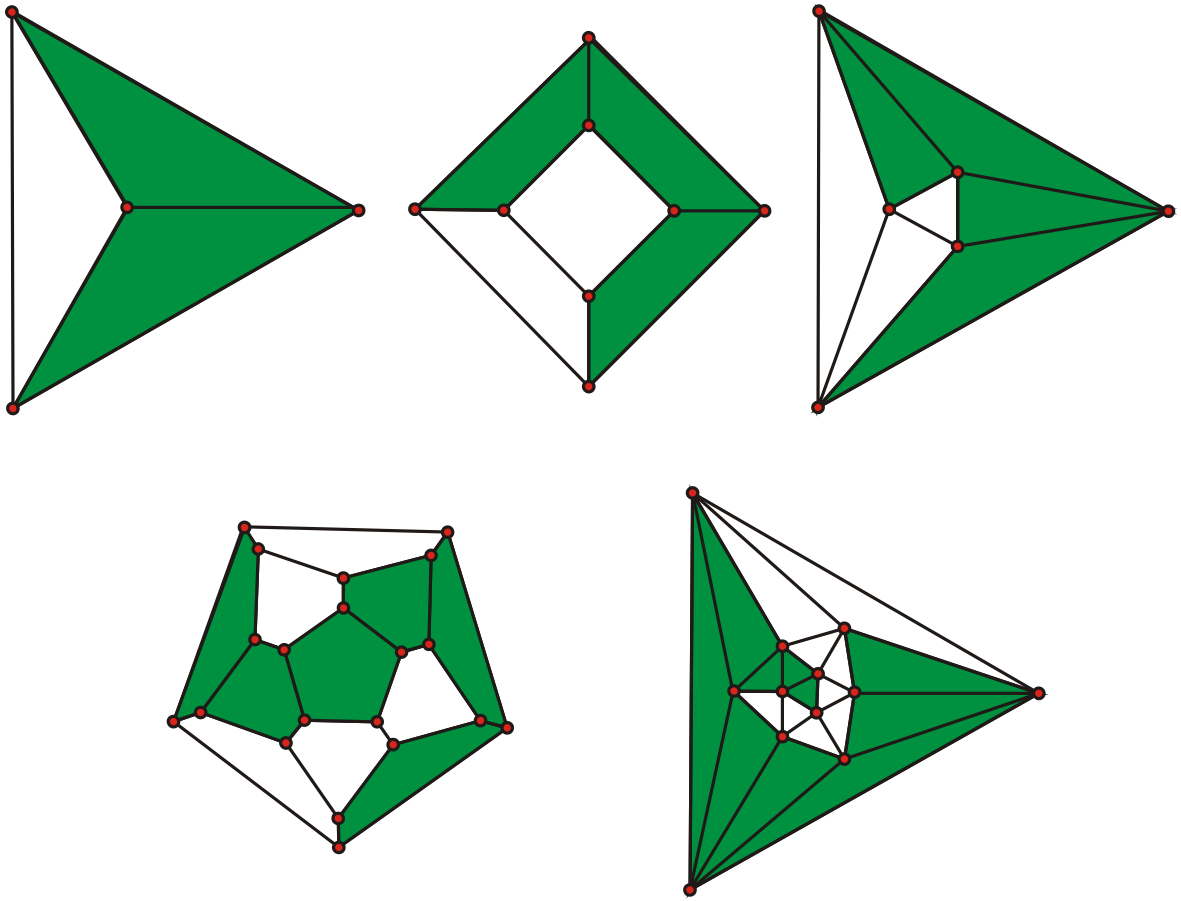
Odprt problem (1969):
Ali ima vsak simetrični graf hamiltonsko pot oziroma hamiltonski cikel?

Petersenov graf

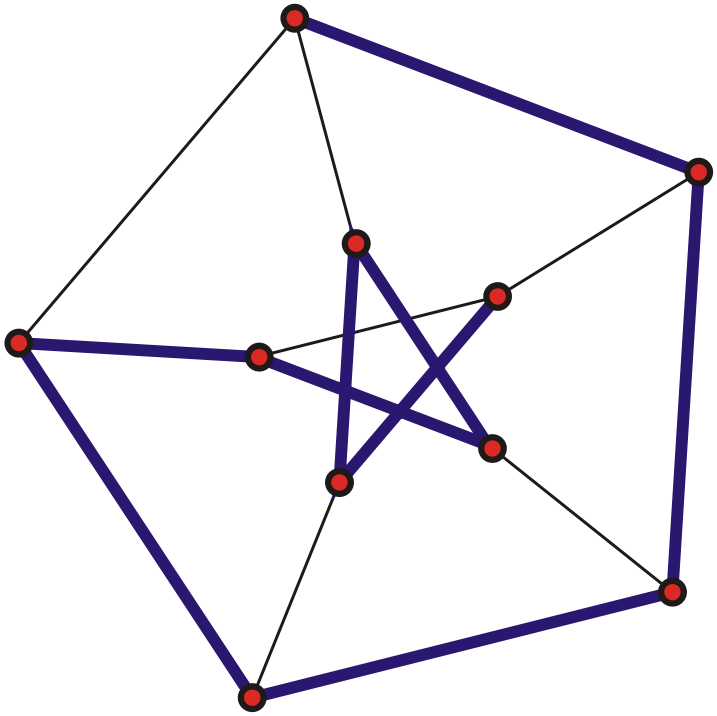




Kako poiskati hamiltonski cikel v ravninskem grafu?



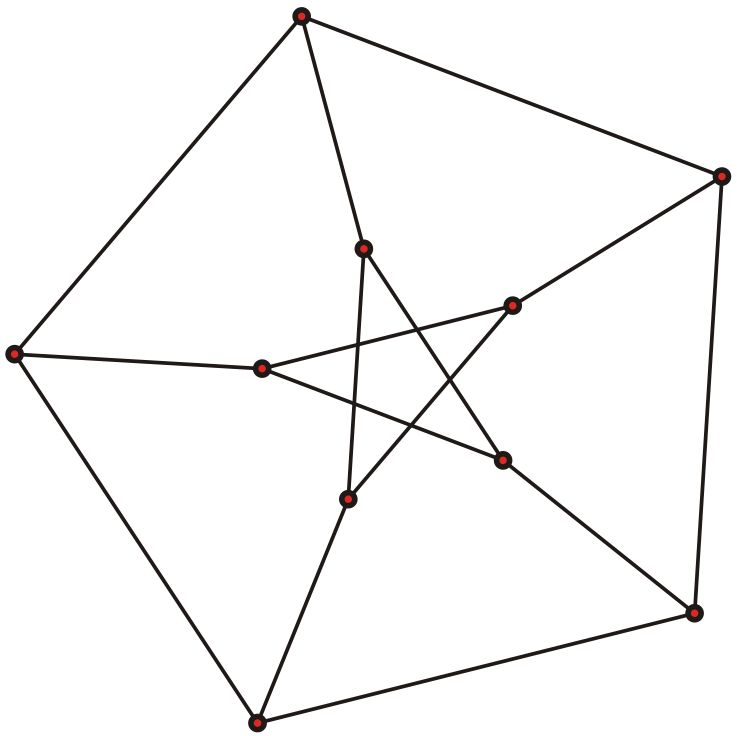
Petersenov graf



Hamiltonska pot

Petersenov graf

Kako pokažemo, da Petersenov graf nima hamiltonskega cikla?



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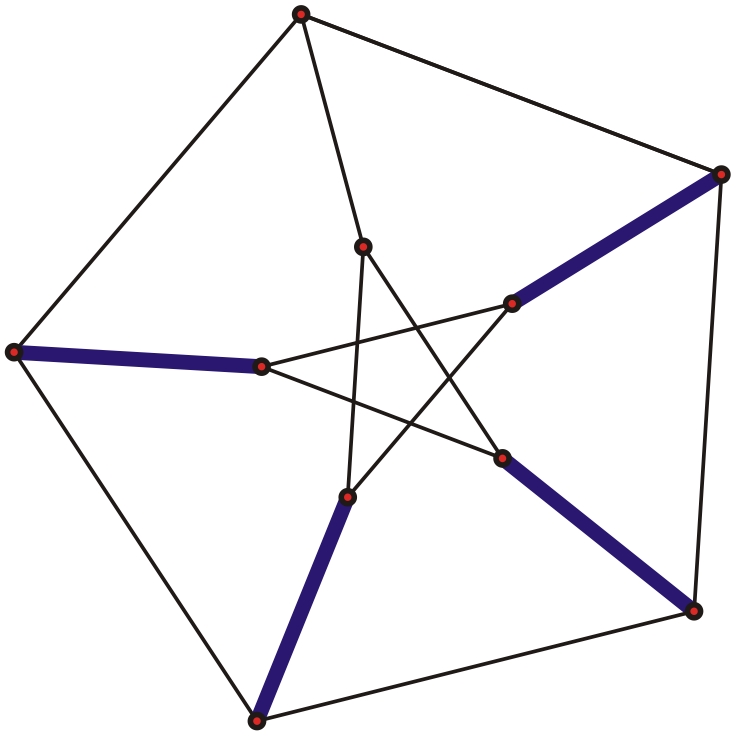

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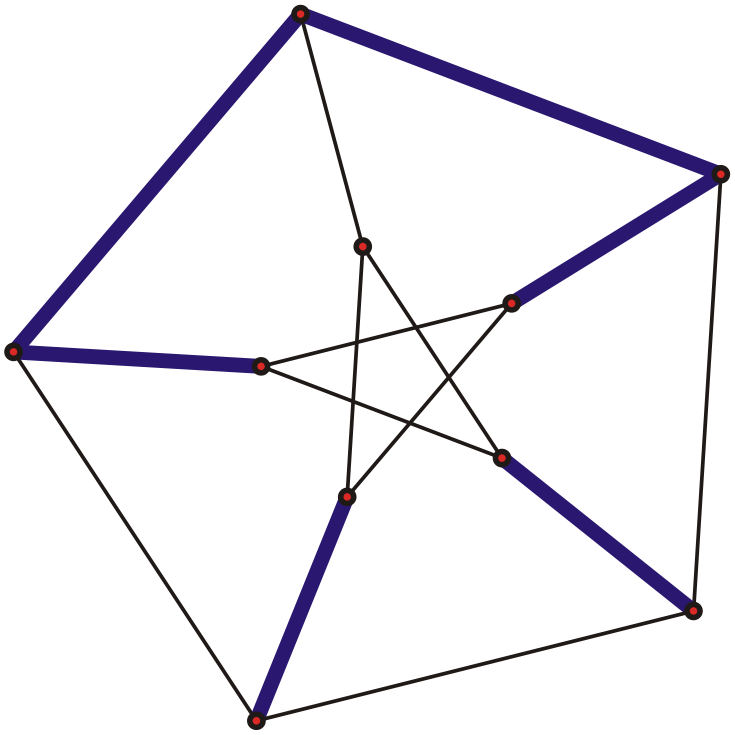

Petersenov graf

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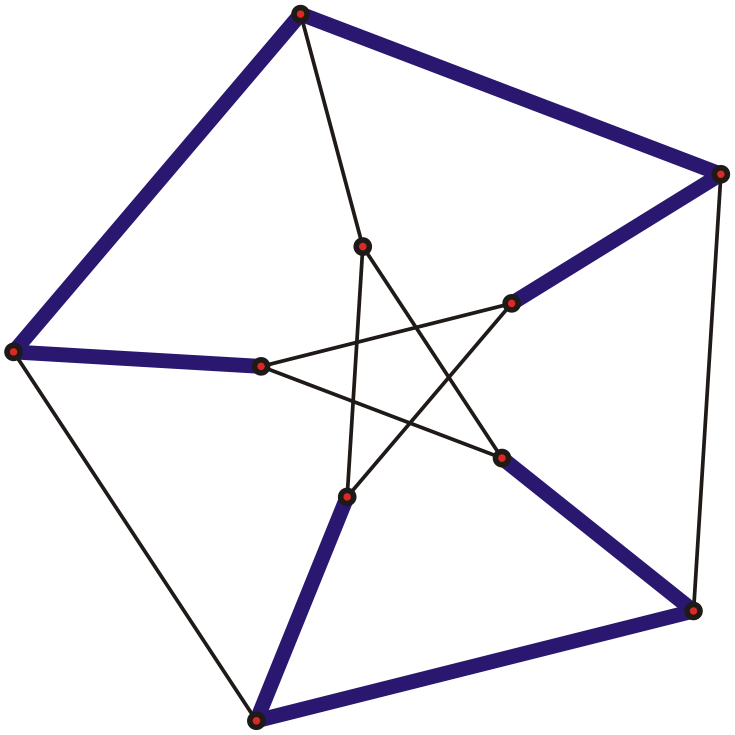

Petersenov graf

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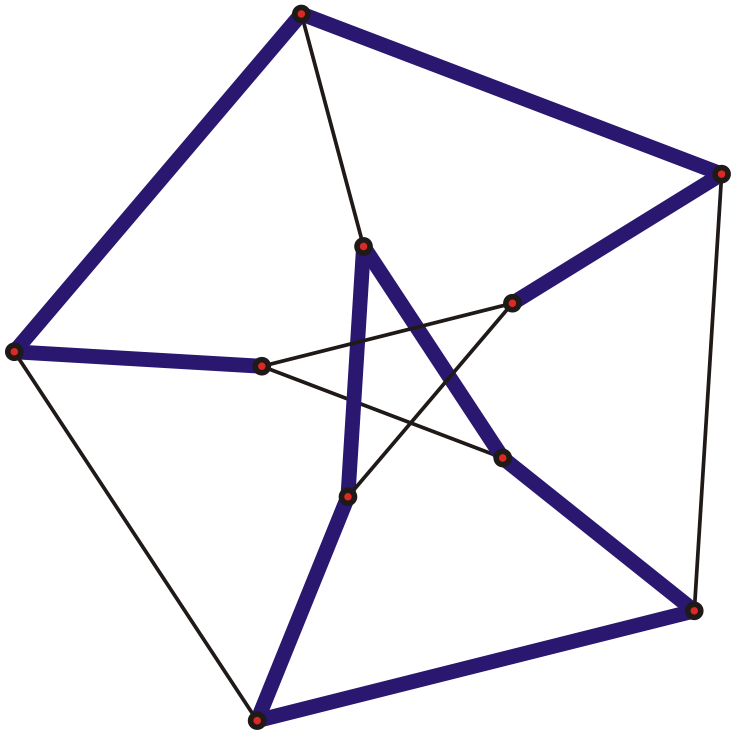

Petersenov graf

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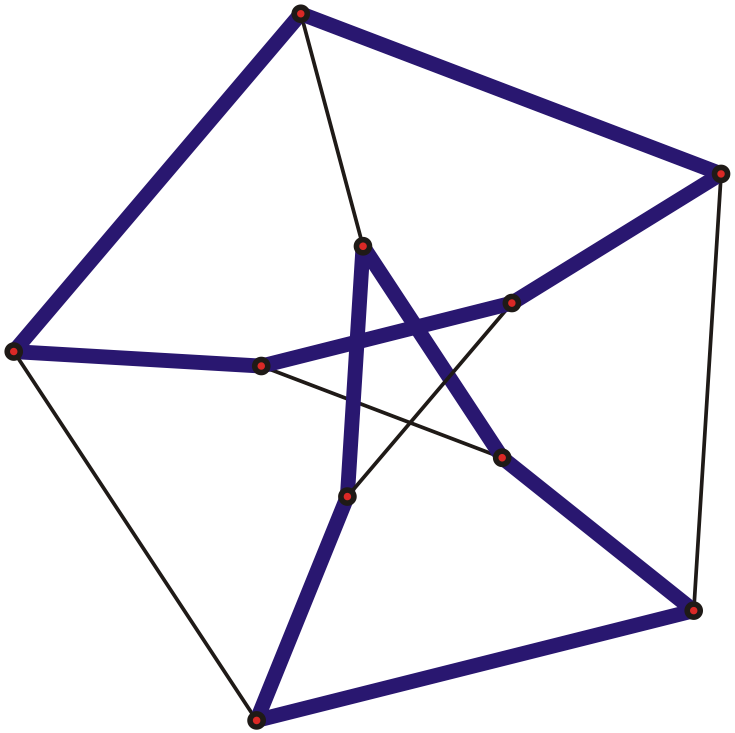
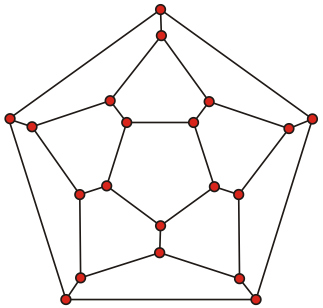
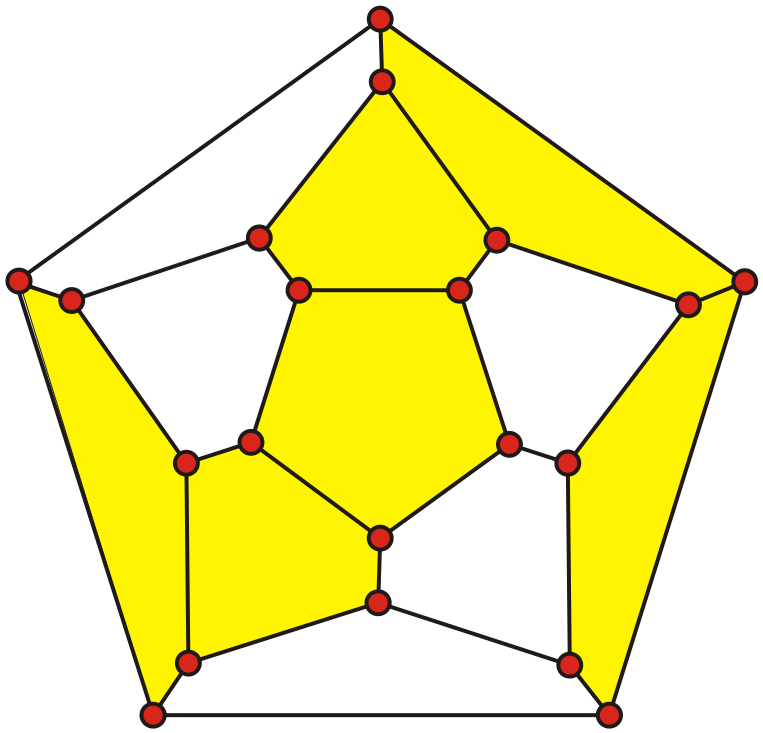


Dodekaeder (Ikozaedrska igra)



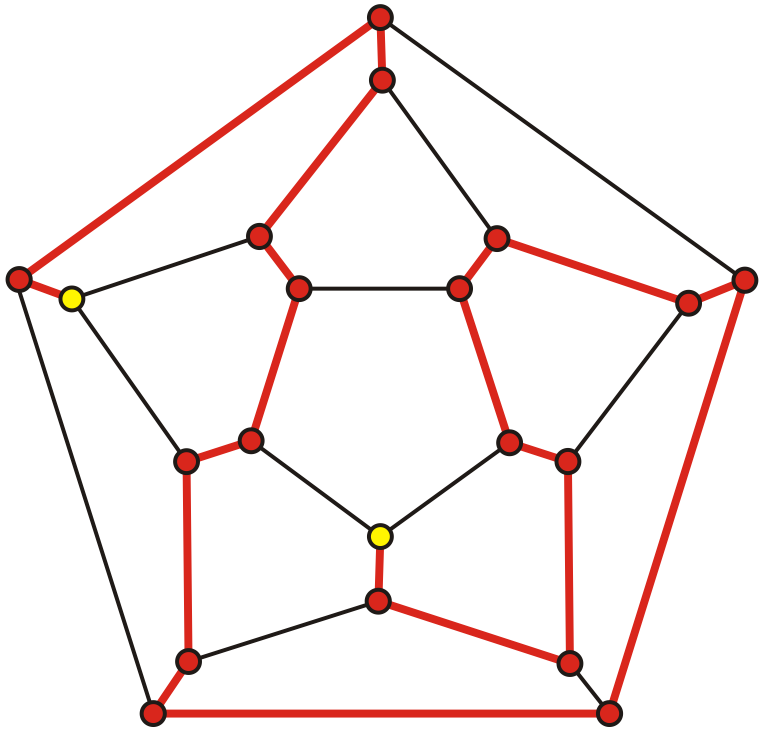
Naj bosta u in v točki dodekaedra. Ali obstaja hamiltonska pot med u in v?

Dodekaeder (Ikozaedrska igra)



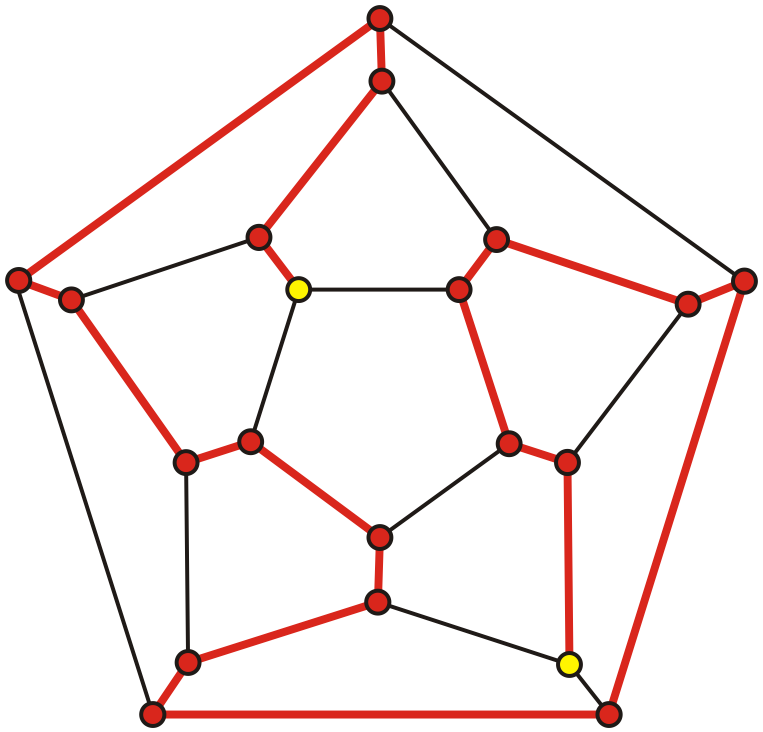
Razdalja 1

Dodekaeder (Ikozaedrska igra)



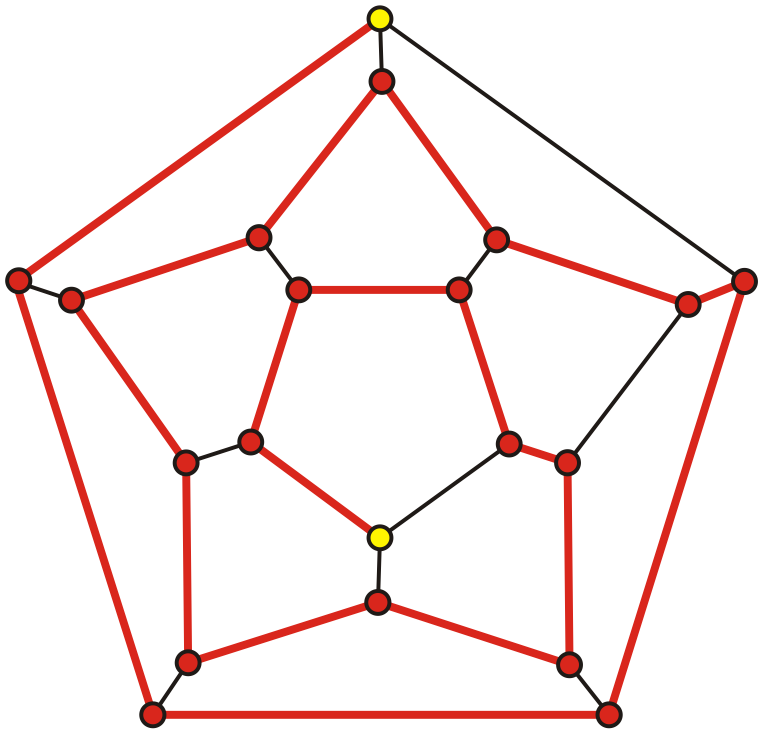
Razdalja 3

Dodekaeder (Ikozaedrska igra)



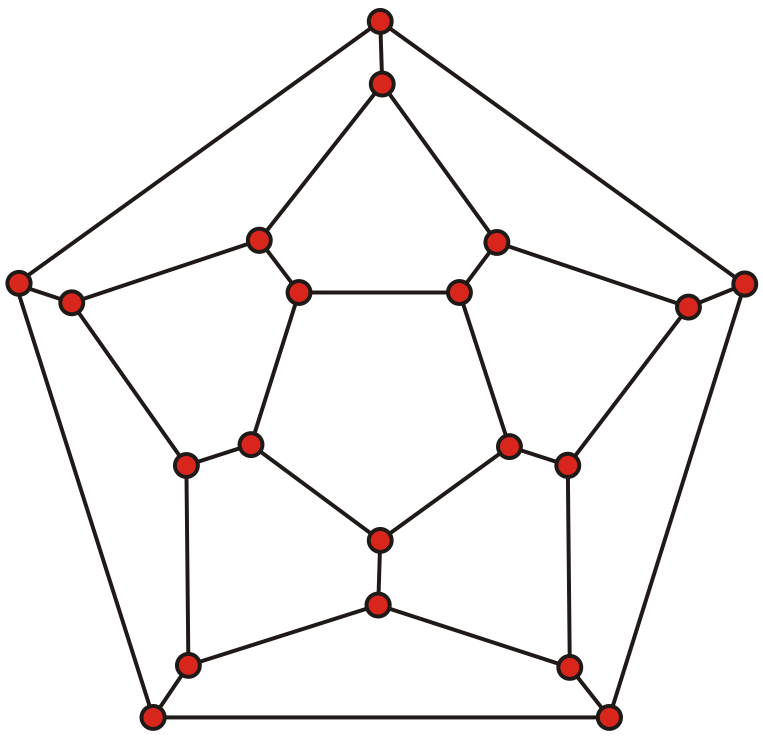
Razdalja 4

Dodekaeder (Ikozaedrska igra)



Razdalja 5

Dodekaeder (Ikozaedrska igra)



Razdalja 2?

Hvala.

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